Non-Fermi liquid corrections to the neutrino mean free path in dense quark matter

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We calculate the neutrino mean free path with non-Fermi liquid (NFL) corrections in quark matter from scattering and absorption processes for both degenerate and nondegenerate neutrinos. We show that the mean free path decreases due to the non-Fermi liquid corrections leading to $l_{mean}^{-1} \sim [...... + ...C_F^2\alpha_s^2 \ln(m_D/T)^2]$. These reduction results in higher rate of scattering leading to faster cooling of stars for nondegenerate neutrinos. For degenerate case the corresponding mean free path is found to be much smaller than the core radius of the neutron star which maintains the local thermodynamic equilibrium.

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I. INTRODUCTION

Recently, there has been a substantial effort to study the properties of cold and warm quark matter. Such studies are important to understand the properties of the astrophysical compact objects like neutron stars and pulsars. There has been lot of experimental efforts like Einstein laboratory, ROSAT, CHANDRA and XMM, where various measurements are performed to understand the properties of neutron stars [1, 2].

There is a possibility that at the core of neutron stars the density may go upto $5 \sim 6$ times the normal nuclear matter density where the matter is not expected to be in the hadronic phase. In fact, under such a scenario one expects that it would be more appropriate to describe the core of such dense stars as degenerate quark matter [1] which is our main interest in the present work.

It is known that the newly born neutron stars cool via the emissions of neutrinos and

antineutrinos within few minutes involving neutron beta decay and its inverse reaction [3–5]. This cooling behavior is related to the transport properties of the medium or thermodynamical quantities like specific heat, entropy etc. All these quantities are in turn related to the neutrino scattering rate or mean free path (MFP).

Such emissions can take place either by the direct [3–5] or by the modified URCA processes [4, 6]. The direct URCA reactions can proceed in neutron rich matter if the ratio of the proton number density to the total baryon number density exceeds a critical value which follows from the energy and momentum conservation properties [7]. In modified URCA process another nucleon catalyzes the reaction to occur under situations where the direct URCA reaction is forbidden.

For quark matter, similar emission processes proceed through the decay of d quark or the scattering of u-d quarks. Analogously these reactions are named as 'quark direct URCA' processes which have been studied in detail by Iwamoto [3, 4]. Our main focus here is to calculate the neutrino MFP, as mentioned earlier, for cold and warm QCD matter. Previously MFP of quark matter was derived in Ref.[3] where the calculations were restricted to the leading order by assuming free Fermi gas interactions. Here we plan to go beyond leading order to include 'plasma' or 'quasi particle' effects arising out of the interacting ground state. This, as we show, gives rise to logarithmic corrections to the MFP in contrast to what one expects from the usual Fermi liquid theory (FLT) [8, 9]. Similar non-Fermi liquid behavior of various thermodynamical quantities have recently drawn significant attention.

For example, in [1, 10], the authors have computed the leading contribution to the interaction part of specific heat and entropy when temperature is much smaller than the chemical potential of quark matter. It has been shown, in case of specific heat, instead of $\mathcal{O}(T^3)$ term, an anomalous $T \ln T$ term appear as corrections to the leading order which becomes dominant for temperature smaller than the Debye mass. This type of nonanalytic terms are known as non-Fermi liquid (NFL) corrections [1, 2, 5, 10–12]. Ref.[11] examined NFL effects in the normal phase of high density QCD matter both using the Dyson-Schwinger equation and the renormalization group theory. In [12] magnetic susceptibility has also been shown to receive correction $\mathcal{O}(T^2 \ln T)$. Being motivated by these series of works we undertake the present investigation to estimate and see the consequences of such effects in the case of neutrino MFP.

In FLT, quarks are treated as quasiparticles and their energy (E) is regarded as a func-

tional of the distribution function [8, 9, 13–16]. FLT is restricted to the low-lying excitations near the Fermi surface, where the lifetime of quasiparticles are long enough. Therefore, it is an important tool to study the properties of nuclear (or quark) matter. In Ref.[17], it has been argued, that the exchange of dynamically screened transverse gluons introduces infrared divergences in the quark self-energies that lead to the breakdown of the Fermi liquid description of cold and dense QCD in perturbation theory. A detailed study of non-Fermi liquid aspects of the normal state was presented in Ref.[2]. There the spectral density, dispersion relation and width of quasiparticles with momenta near the Fermi surface were derived at $T = 0$ by implementing a renormalization group resummation of the leading logarithmic infrared divergences associated with the emission of soft dynamically screened transverse gluons [1, 2].

We have already mentioned that such anomalous corrections are ultimately connected to the absence of magnetic screening of gluons *via.* Landau damping. One such calculations which we find to the most relevant for the present purpose was performed in [5]. It was shown that the emissivity receives logarithmic corrections is enhanced due to non-Fermi liquid effects. It might not be out of context here to recall two important works on neutrino mean free path in QED plasma. One is due to Tubbs and Schramm [18] and the other is done by Lamb and Pethick [19]. In [18], the resultant mean free path was calculated in the neutranized core and just outside the core. It is concluded in [19], that neutrino degeneracy reduces the neutrino mean free path which suggest that neutrino may flow out of the core rather slowly.

In our work we show that the neutrino mean free path receive logarithmic corrections where the dressed gluon propagator is used instead of bare propagator. Infact, we generalize the [4, 18, 19] results by incorporating the NFL corrections for the quark matter. The corrections to the MFP for degenerate and non-degenerate neutrinos, as we shall see, will involve different powers of α_s.

It is well known that in the interior of the star the neutrino absorption and neutrino scattering can take place. These two types of mean free paths are related to different physical phenomenon. Absorption mean free path gives the transport properties, which characterizes the rate of momentum transport. This also enters into the expression for the diffusion coefficient. While, the scattering mean free paths are related to the relaxation time, which characterizes the rate of change of the neutrino distribution function [4]. To

obtain the total mean free path one can write [20],

$$\frac{1}{l_{mean}^{total}} = \frac{1}{l_{mean}^{abs}} + \frac{1}{l_{mean}^{scatt}} \tag{1}$$

II. MEAN FREE PATH

In this section we calculate neutrino mean free paths in quark matter including NFL corrections both for degenerate and nondegenerate neutrinos. When the neutrino chemical potential (μ_ν) is considered to be much larger than the temperature (T), the neutrinos become degenerate and for nondegenerate neutrinos $\mu_\nu \ll T$. In our model the Lagrangian density is described by [3]

$$\mathcal{L}_{Wx}(x) = \frac{G}{\sqrt{2}} l_\mu(x) \mathcal{J}_W^\mu(x) + H.C. \tag{2}$$

where the weak coupling constant is $G \simeq 1.166 \times 10^{-11}$ in MeV units, and l_μ and $\mathcal{J}_W^\mu$ are the lepton and hadron charged weak currents, respectively. The weak currents are

$$l_\mu(x) = \bar{e}\gamma_\mu(1-\gamma_5)\nu_e + \bar{\mu}\gamma_\mu(1-\gamma_5)\nu_\mu + ..., \tag{3}$$

$$\mathcal{J}_W^\mu(x) = \cos\theta_c \bar{u}\gamma^\mu(1-\gamma_5)d + \sin\theta_c \bar{u}\gamma^\mu(1-\gamma_5)s + ...; \tag{4}$$

where θ_c is the Cabibbo angle ($\cos^2\theta_c \simeq 0.948$) [21].

The mean free path is determined by the quark neutrino interaction in dense quark matter *via.* weak processes. We consider the simplest β decay reactions; the absorption process

$$d + \nu_e \to u + e^- \tag{5}$$

and other is its inverse relation

$$u + e^- \to d + \nu_e \tag{6}$$

The neutrino mean free path is related to the total interaction rate due to neutrino emission averaged over the initial quark spins and summed over the final state phase space and spins. It is given by [4]

$$\begin{aligned}\frac{1}{l_{mean}^{abs}(E_\nu, T)} = {} & \frac{g}{2E_\nu}\int\frac{d^3p_d}{(2\pi)^3}\frac{1}{2E_d}\int\frac{d^3p_u}{(2\pi)^3}\frac{1}{2E_u}\int\frac{d^3p_e}{(2\pi)^3}\frac{1}{2E_e}(2\pi)^4\delta^4(P_d+P_\nu-P_u-P_e) \\ & |M|^2\{n(p_d)[1-n(p_u)][1-n(p_e)] - n(p_u)n(p_e)[1-n(p_d)]\},\end{aligned} \tag{7}$$

where, g is the spin and color degeneracy, which in the present case is considered to be 6. Here, E, p and n_p are the energy, momentum and distribution function for the corresponding particle. $|M|^2$ is the squared invariant amplitude averaged over initial d quark spin and summed over final spins of u quark and electron as given by[4]

$$|M|^2 = \frac{1}{2}\sum_{\sigma_u,\sigma_d,\sigma_e}|M_{fi}|^2 = 64G^2\cos^2\theta_c(P_d\cdot P_\nu)(P_u\cdot P_e) \tag{8}$$

Here, we work with the two flavor system as the interaction involving strange quark is Cabibbo suppressed [1, 5].

A. Degenerate Neutrinos

We now consider the case of degenerate neutrinos $i.e.$ when $\mu_\nu \gg T$ or in other words we consider trapped neutrino matter. So in this case both the direct (Eq.5) and inverse (Eq.6) processes can occur and both the terms in Eq.(7) under curly brackets [4] are retained. Consequently, the β equilibrium condition becomes $\mu_d + \mu_\nu = \mu_u + \mu_e$. Neglecting the quark-quark interactions and by using Eqs.(7) and (8), for the mean free path one obtains

$$\begin{aligned}\frac{1}{l_{mean}^{abs,D}} &= \frac{3}{4\pi^5}G^2\cos^2\theta_c\int d^3p_d\int d^3p_u\int d^3p_e(1-\cos\theta_{d\nu})(1-\cos\theta_{ue})\\ &\times\ \delta^4(P_d+P_\nu-P_u-P_e)[1+e^{-\beta(E_\nu-\mu_\nu)}]n(p_d)[1-n(p_u)][1-n(p_e)]\end{aligned} \tag{9}$$

In the square bracket, the second term $e^{-\beta(E_\nu-\mu_\nu)}$ is due to the inverse process (Eq.6). Since the masses of u, d quark and electron are very small, one can neglect the mass effect on the mean free path. To carry out the momentum integration we define $p \equiv |p_d+p_\nu| = |p_u+p_e|$ as a variable. Following the procedure, described by Iwamoto [4] one has

$$\sin\theta_{d\nu}d\theta_{d\nu} = \frac{pdp}{p_f(d)p_f(\nu)} \tag{10}$$

$$(1-\cos\theta_{d\nu})(1-\cos\theta_{ue}) \simeq \frac{p^4-2p^2p_f^2+p_f^4}{4p_f(d)p_f(\nu)p_f(u)p_f(e)} \tag{11}$$

and

$$\begin{aligned}d^3p_d &= 2\pi\sin\theta_{d\nu}d\theta_{d\nu}p_d^2dp_d\\ &= 2\pi\frac{p_f(d)}{p_f(\nu)}pdp\frac{dp_d}{dE_d}dE_d\\ &= 2\pi\frac{p_f(d)}{p_f(\nu)}pdp\frac{dp_d}{d\omega}d\omega\end{aligned} \tag{12}$$

$$d^3p_u = 2\pi\frac{p_f(u)p_f(e)}{p}dE_e\frac{dp_u}{d\omega}d\omega \tag{13}$$

where we denote the single particle energy $E_{d(u)}$ as ω. For the free case $dp/d\omega$ is the inverse quark velocity. It is well known that this slope of the dispersion relation changes in matter due to scattering from the Fermi surface and excitation of the Dirac vacuum. The modified dispersion relation can be obtained by computing the on-shell one-loop self-energy. For quasiparticles with momenta close to the Fermi momentum, the one-loop self-energy is dominated by the soft gluon exchanges [22]. The quasiparticle energy ω satisfies the relation [22, 23]

$$\omega \;=\; E_p(\omega) + \mathrm{Re}\Sigma(\omega, p(\omega)) \tag{14}$$

where we have approximated only to the real part of self energy, since the imaginary part of Σ turns out to be negligible compared to its real part [10, 12]. The detailed analysis can be obtained in [23, 24].

In the relativistic case, when the Fermi velocity v_f is close to the velocity of light c, exchange of magnetic gluons become important. In the non-relativistic case it is suppressed by a factor $(v/c)^2$ with respect to exchange of electric gauge bosons and is usually neglected. The magnetic interaction, as stated before, is screened only dynamically and the problem remains for the static gluons [22, 25]. Therefore to obtain a finite result, a suitable resummation has to be performed [26, 27]. The analytical expressions for one-loop quark self-energy can be written as [10–12, 17, 22]

$$\Sigma \;=\; \frac{g^2 C_F}{12\pi^2}(\omega-\mu)\ln\left(\frac{m_D}{\omega-\mu}\right) + i\frac{g^2 C_F}{12\pi}|\omega-\mu|, \tag{15}$$

It exhibits a logarithmic singularity close to the Fermi surface *i.e.* when $\omega \to \mu$. Thus the long ranged character of the magnetic interactions spoils the normal Fermi-liquid behavior [8, 9, 13–16]. The breakdown of the Fermi liquid picture is associated with the vanishing of the discontinuity of the distribution function at the Fermi surface [2, 17]. This non-perturbative nature of the self energy gives rise to the non-Fermi liquid behavior. Here, m_D is a cut-off factor and should be an order of the Debye mass. Differentiating Eq.(14) with respect to p, we obtain $\frac{dp(\omega)}{d\omega}$ at leading order in $\frac{T}{\mu}$ is

$$\begin{aligned} \frac{dp(\omega)}{d\omega} &\simeq \left(1 - \frac{\partial}{\partial\omega}\mathrm{Re}\Sigma(\omega)\right)\frac{E_p(\omega)}{p(\omega)} \\ &= \left[1 + \frac{C_F\alpha_s}{3\pi}\ln\left(\frac{m_D}{T}\right)\right]\frac{E_p(\omega)}{p(\omega)} \end{aligned} \tag{16}$$

where α_s is the strong coupling constant, $C_F = (N_c^2 - 1)/(2N_c)$ and N_c is the color factor. Using Eqs.(16), (9) and (11-13), the neutrino mean free path can be determined for two conditions. For $|p_f(u) - p_f(e)| \geq |p_f(d) - p_f(\nu)|$

$$\begin{aligned}\frac{1}{l_{mean}^{abs,D}} &= \frac{4}{\pi^3}G^2\cos^2\theta_c\frac{\mu_u^2\mu_e^3}{\mu_\nu^2}\Big[1+\frac{1}{2}\Big(\frac{\mu_e}{\mu_u}\Big)+\frac{1}{10}\Big(\frac{\mu_e}{\mu_u}\Big)^2\Big]\\ &\times\ [(E_\nu-\mu_\nu)^2+\pi^2T^2]\Big[1+\frac{C_F\alpha_s}{3\pi}\ln\Big(\frac{m_D}{T}\Big)\Big]^2 \qquad (17)\end{aligned}$$

To derive above Eq.(17), we use the result of integral [4, 28]

$$\begin{aligned}&\int_0^\infty dE_d\int_0^\infty dE_u\int_0^\infty dE_e[1+e^{-\beta(E_\nu-\mu_\nu)}]n(p_d)[1-n(p_u)][1-n(p_e)]\delta(E_d+E_\nu-E_u-E_e)\\ &\simeq\frac{1}{2}[(E_\nu-\mu_\nu)^2+\pi^2T^2] \qquad (18)\end{aligned}$$

Similarly, for $|p_f(d) - p_f(\nu)| \geq |p_f(u) - p_f(e)|$, the corresponding expression for mean free path can be obtained by replacing $\mu_u \leftrightarrow \mu_d$ and $\mu_e \leftrightarrow \mu_\nu$ in Eq.(17). Since quark and electron are assumed to be massless, the chemical equilibrium condition gives $p_f(u)+p_f(e) = p_f(d) + p_f(\nu)$, which we use to derive Eq.(17).

The other major contribution to the mean free path arises from quark-neutrino scattering. The neutrino scattering process from degenerate quarks are given by

$$q_i + \nu_e(\overline{\nu}_e) \rightarrow q_i + \nu_e(\overline{\nu}_e) \qquad (19)$$

for each quark component of flavor $i(= u$ or $d)$. The scattering mean free path of the neutrinos in degenerate case can be calculated similarly as evaluated by Lamb and Pethick in [19] for electron-neutrino scattering. Assuming $m_{q_i}/p_{f_i} \ll 1$ and including the non-Fermi liquid correction through phase space, the mean free path is given by

$$\frac{1}{l_{mean}^{scatt,D}} = \frac{3}{16}n_{q_i}\sigma_0\Big[\frac{(E_\nu-\mu_\nu)^2+\pi^2T^2}{m_{q_i}^2}\Big]\Big[1+\frac{C_F\alpha_s}{3\pi}\ln\Big(\frac{m_D}{T}\Big)\Big]^2\Lambda(x_i) \qquad (20)$$

Here, m_{q_i} is the quark mass. C_{V_i} and C_{A_i} is the vector and axial vector coupling constant was given in TABLE-II of Ref.[4]. In Eq.(20) if we drop the color factor and the second square bracketed term, we obtain results reported in [19] for dense and cold QED plasma with m_q replaced by m_e. In Eq.(20) the constants $\sigma_0 \equiv 4G^2m_{q_i}^2/\pi$ [18] and n_{q_i} is the number density of quark, is given by

$$n_{q_i} = 2\int\frac{d^3p}{(2\pi)^3}\frac{1}{e^{\beta(E_{q_i}-\mu_{q_i})}} \qquad (21)$$

where 2 is the quark spin degeneracy factor. Explicit form of $\Lambda(x_i)$ can be written as [4, 19]

$$\Lambda(x_i) \;=\; \frac{4}{3}\frac{\text{Min}(\mu_\nu,\mu_{q_i})}{\mu_{q_i}}\Big[(C_{V_i}^2 + C_{A_i}^2)\Big(2+\frac{1}{5}x_i^2\Big) + 2C_{V_i}C_{A_i}x_i\Big] \tag{22}$$

and $x_i = \mu_\nu/\mu_{q_i}$ if $\mu_\nu < \mu_{q_i}$ and $x_i = \mu_{q_i}/\mu_\nu$ if $\mu_\nu > \mu_{q_i}$.

B. Nondegenerate Neutrinos

We also derive mean free path for nondegenerate neutrinos *i.e.* when $\mu_\nu \ll T$. For nondegenerate neutrinos the inverse process (6) is dropped. Hence we neglect the second term in the curly braces of Eq.(7). In this case, only those fermions whose momenta lie close to their respective Fermi surfaces can take part in a reaction. It is to be mentioned here, if quarks are treated as free, as discussed in [4, 29, 30], the matrix element vanishes, since u, d quarks and electrons are collinear in momenta. The inclusion of strong interactions between quarks relaxes these kinematic restrictions resulting in a nonvanishing squared matrix amplitude. Since the neutrinos are produced thermally, we neglect the neutrino momentum in energy-momentum conservation relation [4]. This is not the case for degenerate neutrinos where $p_\nu \gg T$ and therefore such approximation is not valid there. By doing angular average over the direction of the outgoing neutrino, from Eq.(8) the squared matrix element is given by [5]

$$\begin{aligned}|M|^2 \;&=\; 64G^2\cos^2\theta_c p_f^2 (V_d\cdot P_\nu)(V_u\cdot P_e)\\ &=\; 64G^2\cos^2\theta_c p_f^2 E_\nu \mu_e \frac{C_F\alpha_s}{\pi}\end{aligned} \tag{23}$$

where $V = (1, v_f)$ is the four velocity. To calculate Eq.(23) we have used the chemical equilibrium condition $\mu_d = \mu_u + \mu_e$ and also the relations derived from Fermi liquid theory are given by[5]

$$v_F = 1 - \frac{C_F\alpha_s}{2\pi}\;; \qquad \delta\mu = \frac{C_F\alpha_s}{\pi}\mu \tag{24}$$

Putting $|M|^2$ in Eq.(7) we have,

$$\begin{aligned}\frac{1}{l_{mean}^{abs,ND}} \;&=\; \frac{3C_F\alpha_s}{4\pi^6}G^2\cos^2\theta_c\int d^3p_d\int d^3p_u\int d^3p_e\\ &\quad\delta^4(P_d + P_\nu - P_u - P_e)n(p_d)[1-n(p_u)][1-n(p_e)]\end{aligned} \tag{25}$$

Neglecting the neutrino momentum in the neutrino momentum conserving δ function, the integrals can be decoupled into two parts. Following the procedure of Iwamoto [4], the angular integral is given by

$$\mathcal{A} \;=\; \int d\Omega_d \int d\Omega_u \int d\Omega_e \delta(p_d - p_u - p_e) \;=\; \frac{8\pi^2}{\mu_d \mu_u \mu_e} \tag{26}$$

and the other part

$$\begin{aligned} \mathcal{B} \;=\; & \int_0^\infty p_d^2 \frac{dp_d}{dE_d} dE_d \int_0^\infty p_u^2 \frac{dp_u}{dE_u} dE_u \int_0^\infty p_e^2 dE_e \\ & \delta(E_d + E_\nu - E_u - E_e) n(p_d)[1 - n(p_u)][1 - n(p_e)] \end{aligned} \tag{27}$$

Changing the variables to $x_d = (E_d - \mu_d)\beta$, $x_u = -(E_u - \mu_u)\beta$ and $x_e = -(E_e - \mu_e)\beta$ and denoting the single particle energy $E_{u(d)}$ as ω we have from Eq.(27)

$$\mathcal{B} = \int_{-\infty}^{\infty} dx_d dx_u dx_e \frac{dp_d(\omega)}{d\omega} \frac{dp_u(\omega)}{d\omega} p_d^2 p_u^2 p_e^2 \delta(x_d + x_u + x_e + \beta E_\nu) n(x_d) n(-x_u) n(-x_e) \tag{28}$$

As the contribution dominates near the Fermi surfaces, extension of lower limit is a reasonable approximation [28, 31].

Using Eq.(16) and perform the integration of Eq.(28) following the procedure defined in [12, 28, 31], we have

$$\mathcal{B} \;=\; \mu_d^2 \mu_u^2 \mu_e^2 \, \frac{(E_\nu^2 + \pi^2 T^2)}{2(1 + e^{-\beta E_\nu})} \left[1 + \frac{C_F \alpha_s}{3\pi} \ln\left(\frac{m_D}{T}\right)\right]^2 \tag{29}$$

Using Eqs.(25), (26) and (29), the mean free path at leading order in T/μ is given by

$$\frac{1}{l_{mean}^{abs,ND}} \;=\; \frac{3 C_F \alpha_s}{\pi^4} G^2 \cos^2\theta_c \; \mu_d \; \mu_u \; \mu_e \; \frac{(E_\nu^2 + \pi^2 T^2)}{(1 + e^{-\beta E_\nu})} \left[1 + \frac{C_F \alpha_s}{3\pi} \ln\left(\frac{m_D}{T}\right)\right]^2 \tag{30}$$

The first term is known from [4] and the additional terms are higher order corrections to the previous results derived in the present work.

For the scattering of nondegenerate neutrinos in quark matter, the expression of mean free path was given by Iwamoto[4]. We incorporate the anomalous effect which enters through phase space modification giving rise to

$$\frac{1}{l_{mean}^{scatt,ND}} \;=\; \frac{C_{V_i}^2 + C_{A_i}^2}{20} n_{q_i} \sigma_0 \left(\frac{E_\nu}{m_{q_i}}\right)^2 \left(\frac{E_\nu}{\mu_i}\right) \left[1 + \frac{C_F \alpha_s}{3\pi} \ln\left(\frac{m_D}{T}\right)\right]^2. \tag{31}$$

Here, we have assumed $m_{q_i}/p_{f_i} \ll 1$ and the constants σ_0 and number density n_{q_i} defined earlier.

TABLE I: Comparison of neutrino mean free path in quark matter for the case of degenerate and nondegenerate neutrinos. Mean free path given in unit of meter, without NFL correction and including NFL correction for the case of absorption and scattering processes.

	Absorption		Scattering	
	Without Correction	With Correction	Without Correction	With Correction
Degenerate	3.62	1.85	15.88	8.13
Nondegenerate	7.68	3.93	1585.18	811.06

III. SUMMARY AND CONCLUSION

In this work we have calculated, the neutrino mean free path with NFL corrections in quark matter for both degenerate and nondegenerate neutrinos. Contribution to the total mean free path arises from absorption and scattering processes. We find, for absorption processes, corrections to the MFP is $\mathcal{O}(\alpha_s^2)$ for degenerate neutrinos and for non-degenerate case it is $\mathcal{O}(\alpha_s^3)$. This is due to the fact, that for non-degenerate neutrinos, quarks and electrons are collinear in momenta on their Fermi surfaces, for which phase space for direct URCA processes vanishes. To get nonvanishing phase space which requires that the Fermi momenta of quarks and electrons should satisfy triangular relation, one must take into account the interaction between the quarks.

Armed with the results of the previous sections, we now estimate the numerical values of the neutrino mean free paths in quark matter both for degenerate and nondegenerate neutrinos. The mean free paths are listed in Table-I in unit of meter where the neutrino energy (E_ν) is set to be equal to the temperature (T) and $m_q = 10$ MeV[4]. Following ref.[5] we take the quark chemical potential $\mu_q \simeq 500$ MeV corresponding to densities $\rho_b \approx 6\rho_0$, ρ_0 is the nuclear matter saturation density. The electron chemical potential is determined by using the charge neutrality and beta equilibrium conditions which yields $\mu_e = 11$ Mev. The other parameter is used here are same as [5].

From Table-I, we see that the anomalous logarithmic terms dominate the mean free path and reduces its value considerably. This reduced mean free path results in higher rate of scattering leading to higher rate of cooling of the compact stars for nondegenerate neutrinos.

We find the relatively short neutrino mean free path for degenerate neutrinos. These suggest that conditions are favorable for building up of the neutrino number in the core and the neutrino distribution function may approach close to that required for the enhancement of local thermodynamic equilibrium.

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Non-Fermi liquid corrections to the neutrino mean free path in dense quark matter

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We calculate the neutrino mean free path with non-Fermi liquid (NFL) corrections in quark matter from scattering and absorption processes for both degenerate and nondegenerate neutrinos. We show that the mean free path decreases due to the non-Fermi liquid corrections leading to $l_{mean}^{-1} \sim [...... + ...C_F^2 \alpha_s^2 \ln(m_D/T)^2]$. These reduction results in higher rate of scattering.

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I. INTRODUCTION

Recently, there has been a substantial effort to study the properties of cold and warm quark matter. Such studies are important to understand the properties of the astrophysical compact objects like neutron stars and pulsars. There has been lot of experimental efforts like Einstein laboratory, ROSAT, CHANDRA and XMM, where various measurements are performed to understand the properties of neutron stars [1–3] .

There is a possibility that at the core of neutron stars the density may go upto $5 \sim 6$ times the normal nuclear matter density where the matter is not expected to be in the hadronic phase [4, 5]. In fact, under such a scenario one expects that it would be more appropriate to describe the core of such dense stars as degenerate quark matter [6] which is our main interest in the present work.

It is known that the newly born neutron stars cool via the emissions of neutrinos and antineutrinos within few minutes involving the direct [7–9] or the modified URCA processes [8, 10]. The direct URCA reactions can proceed in neutron rich matter if the ratio of the proton number density to the total baryon number density exceeds a critical value which

follows from the energy and momentum conservation properties [11]. In modified URCA process another nucleon catalyzes the reaction to occur under situations where the direct URCA reaction is forbidden.

For quark matter, the dominant contribution to the emission of neutrinos is given by the quark analogus of β decay and the electron capture [9]. These reactions are named as 'quark direct URCA' processes which have been studied in detail by Iwamoto [7, 8]. Our main focus here is to calculate the neutrino MFP, as mentioned earlier, for cold and warm QCD matter. Previously MFP of quark matter was derived in Ref.[7] where the calculations were restricted to the leading order by assuming free Fermi gas interactions. Here we plan to go beyond leading order to include 'plasma' or 'quasi particle' effects arising out of the interacting ground state. This, as we show, gives rise to logarithmic corrections to the MFP in contrast to what one expects from the usual Fermi liquid theory (FLT) [12, 13]. Similar non-Fermi liquid behavior of various thermodynamical quantities have recently drawn significant attention [4, 5, 9, 14–16].

For example, in [4, 14], the authors have computed the leading contribution to the interaction part of specific heat and entropy when temperature is much smaller than the chemical potential of quark matter. It has been shown, in case of specific heat, for ideal gas the leading order term goes as $\mathcal{O}(\mu^2 T)$, while the correction term involve $\mathcal{O}(T^3)$. In case of interacting case, as shown in ref.[14], the correction to the leading order contribution involve nonanalytic terms, known as non-Fermi liquid (NFL) corrections, which have been discussed extensively in [4, 5, 9, 14–16]. Ref.[15] examined NFL effects in the normal phase of high density QCD matter both using the Dyson-Schwinger equation and the renormalization group theory. Like specific heat, the magnetic susceptibility shows similar non-Fermi liquid behavior has been shown in [16]. Being motivated by these series of works we undertake the present investigation to estimate and see the consequences of such effects in the case of neutrino MFP.

In FLT, quarks are treated as quasiparticles and their energy (E) is regarded as a functional of the distribution function [12, 13, 17–20]. FLT is restricted to the low-lying excitations near the Fermi surface, where the lifetime of quasiparticles are long enough. Therefore, it is an important tool to study the properties of nuclear (or quark) matter. In Ref.[21], it has been argued, that the exchange of dynamically screened transverse gluons introduces infrared divergences in the quark self-energies that lead to the breakdown of the Fermi liquid

description of cold and dense QCD in perturbation theory. A detailed study of non-Fermi liquid aspects of the normal state was presented in Ref.[5]. There the spectral density, dispersion relation and width of quasiparticles with momenta near the Fermi surface were derived at $T = 0$ by implementing a renormalization group resummation of the leading logarithmic infrared divergences associated with the emission of soft dynamically screened transverse gluons [4, 5].

We have already mentioned that such anomalous corrections are ultimately connected to the absence of magnetic screening of gluons *via.* Landau damping. One such calculations which we find to the most relevant for the present purpose was performed in [9]. It was shown that the emissivity receives logarithmic corrections is enhanced due to non-Fermi liquid effects. It might not be out of context here to recall two important works on neutrino mean free path in QED plasma. One is due to Tubbs and Schramm [22] and the other is done by Lamb and Pethick [23]. In [22], the resultant mean free path was calculated in the neutranized core and just outside the core. It is concluded in [23], that neutrino degeneracy reduces the neutrino mean free path which suggest that neutrino may flow out of the core rather slowly.

In our work we show that the neutrino mean free path receive logarithmic corrections where the dressed gluon propagator is used instead of bare propagator. Infact, we generalize the [8, 22, 23] results by incorporating the NFL corrections for the quark matter. The corrections to the MFP for degenerate and non-degenerate neutrinos, as we shall see, will involve different powers of α_s.

In the interior of a neutron star, there are two distinct phenomenon for which the neutrino mean free path is calculated, one is absorption and the other one involves scattering of neutrinos [8]. The corresponding mean free paths are denoted by l^{abs}_{mean} and l^{scatt}_{mean}. It is to be noted that one also defines another MFP, known as transport mean free path which enters into the calculation of diffusion coefficient. The scattering MFP, on the other hand is related to the relaxation time that characterizes the rate of change of the neutrino distribution function [8]. One can combine l^{scatt}_{mean} with l^{abs}_{mean} to define total mean free path [24],

$$\frac{1}{l^{total}_{mean}} = \frac{1}{l^{abs}_{mean}} + \frac{1}{l^{scatt}_{mean}}. \tag{1}$$

II. MEAN FREE PATH

In this section we calculate neutrino mean free paths in quark matter including NFL corrections both for degenerate and nondegenerate neutrinos. When the neutrino chemical potential (μ_ν) is considered to be much larger than the temperature (T), the neutrinos become degenerate and for nondegenerate neutrinos $\mu_\nu \ll T$. In our model the Lagrangian density is described by [7]

$$\mathcal{L}_{Wx}(x) = \frac{G}{\sqrt{2}} l_\mu(x) \mathcal{J}_W^\mu(x) + H.C. \tag{2}$$

where the weak coupling constant is $G \simeq 1.166 \times 10^{-11}$ in MeV units, and l_μ and $\mathcal{J}_W^\mu$ are the lepton and hadron charged weak currents, respectively. The weak currents are

$$l_\mu(x) = \bar{e}\gamma_\mu(1-\gamma_5)\nu_e + \bar{\mu}\gamma_\mu(1-\gamma_5)\nu_\mu + ..., \tag{3}$$

$$\mathcal{J}_W^\mu(x) = \cos\theta_c \bar{u}\gamma^\mu(1-\gamma_5)d + \sin\theta_c \bar{u}\gamma^\mu(1-\gamma_5)s + ...; \tag{4}$$

where θ_c is the Cabibbo angle ($\cos^2\theta_c \simeq 0.948$) [25].

The mean free path is determined by the quark neutrino interaction in dense quark matter *via.* weak processes. We consider the simplest β decay reactions; the absorption process

$$d + \nu_e \to u + e^- \tag{5}$$

and other is its inverse relation

$$u + e^- \to d + \nu_e \tag{6}$$

The neutrino mean free path is related to the total interaction rate due to neutrino emission averaged over the initial quark spins and summed over the final state phase space and spins. It is given by [8]

$$\begin{aligned}\frac{1}{l_{mean}^{abs}(E_\nu, T)} = {} & \frac{g}{2E_\nu}\int\frac{d^3p_d}{(2\pi)^3}\frac{1}{2E_d}\int\frac{d^3p_u}{(2\pi)^3}\frac{1}{2E_u}\int\frac{d^3p_e}{(2\pi)^3}\frac{1}{2E_e}(2\pi)^4\delta^4(P_d+P_\nu-P_u-P_e) \\ & |M|^2\{n(p_d)[1-n(p_u)][1-n(p_e)] - n(p_u)n(p_e)[1-n(p_d)]\},\end{aligned} \tag{7}$$

where, g is the spin and color degeneracy, which in the present case is considered to be 6. Here, E, p and n_p are the energy, momentum and distribution function for the corresponding

particle. $|M|^2$ is the squared invariant amplitude averaged over initial d quark spin and summed over final spins of u quark and electron as given by[8]

$$|M|^2 = \frac{1}{2}\sum_{\sigma_u,\sigma_d,\sigma_e}|M_{fi}|^2 = 64G^2\cos^2\theta_c(P_d\cdot P_\nu)(P_u\cdot P_e) \tag{8}$$

Here, we work with the two flavor system as the interaction involving strange quark is Cabibbo suppressed [4, 9].

A. Degenerate Neutrinos

We now consider the case of degenerate neutrinos *i.e.* when $\mu_\nu \gg T$ or in other words we consider trapped neutrino matter. So in this case both the direct (Eq.5) and inverse (Eq.6) processes can occur and both the terms in Eq.(7) under curly brackets [8] are retained. Consequently, the β equilibrium condition becomes $\mu_d + \mu_\nu = \mu_u + \mu_e$. Neglecting the quark-quark interactions and by using Eqs.(7) and (8), for the mean free path one obtains

$$\begin{aligned}\frac{1}{l_{mean}^{abs,D}} &= \frac{3}{4\pi^5}G^2\cos^2\theta_c\int d^3p_d\int d^3p_u\int d^3p_e(1-\cos\theta_{d\nu})(1-\cos\theta_{ue})\\ &\times\ \delta^4(P_d+P_\nu-P_u-P_e)[1+e^{-\beta(E_\nu-\mu_\nu)}]n(p_d)[1-n(p_u)][1-n(p_e)]\end{aligned} \tag{9}$$

In the square bracket, the second term $e^{-\beta(E_\nu-\mu_\nu)}$ is due to the inverse process (Eq.6). Since the masses of u, d quark and electron are very small, one can neglect the mass effect on the mean free path. To carry out the momentum integration we define $p \equiv |p_d + p_\nu| = |p_u + p_e|$ as a variable. Following the procedure, described by Iwamoto [8] one has

$$\sin\theta_{d\nu}d\theta_{d\nu} = \frac{pdp}{p_f(d)p_f(\nu)} \tag{10}$$

$$(1-\cos\theta_{d\nu})(1-\cos\theta_{ue}) \simeq \frac{p^4-2p^2p_f^2+p_f^4}{4p_f(d)p_f(\nu)p_f(u)p_f(e)} \tag{11}$$

and

$$\begin{aligned}d^3p_d &= 2\pi\sin\theta_{d\nu}d\theta_{d\nu}p_d^2dp_d\\ &= 2\pi\frac{p_f(d)}{p_f(\nu)}pdp\frac{dp_d}{dE_d}dE_d\\ &= 2\pi\frac{p_f(d)}{p_f(\nu)}pdp\frac{dp_d}{d\omega}d\omega\end{aligned} \tag{12}$$

$$d^3p_u = 2\pi\frac{p_f(u)p_f(e)}{p}dE_e\frac{dp_u}{d\omega}d\omega \tag{13}$$

where we denote the single particle energy $E_{d(u)}$ as ω. For the free case $dp/d\omega$ is the inverse quark velocity. It is well known that this slope of the dispersion relation changes in matter due to scattering from the Fermi surface and excitation of the Dirac vacuum. The modified dispersion relation can be obtained by computing the on-shell one-loop self-energy. For quasiparticles with momenta close to the Fermi momentum, the one-loop self-energy is dominated by the soft gluon exchanges [26]. The quasiparticle energy ω satisfies the relation [26, 27]

$$\omega \; = \; E_p(\omega) + \mathrm{Re}\Sigma(\omega, p(\omega)) \tag{14}$$

where we have approximated only to the real part of self energy, since the imaginary part of Σ turns out to be negligible compared to its real part [14, 16]. The detailed analysis can be obtained in [27, 28].

In the relativistic case, when the Fermi velocity v_f is close to the velocity of light c, exchange of magnetic gluons become important. In the non-relativistic case it is suppressed by a factor $(v/c)^2$ with respect to exchange of electric gauge bosons and is usually neglected. The magnetic interaction, as stated before, is screened only dynamically and the problem remains for the static gluons [26, 29]. Therefore to obtain a finite result, a suitable resummation has to be performed [30, 31]. The analytical expressions for one-loop quark self-energy can be written as [14–16, 21, 26]

$$\Sigma \; = \; \frac{g^2 C_F}{12\pi^2}(\omega-\mu)\ln\left(\frac{m_D}{\omega-\mu}\right) + i\frac{g^2 C_F}{12\pi}|\omega-\mu|, \tag{15}$$

It exhibits a logarithmic singularity close to the Fermi surface *i.e.* when $\omega \to \mu$. Thus the long ranged character of the magnetic interactions spoils the normal Fermi-liquid behavior [12, 13, 17–20]. The breakdown of the Fermi liquid picture is associated with the vanishing of the discontinuity of the distribution function at the Fermi surface [5, 21]. This non-perturbative nature of the self energy gives rise to the non-Fermi liquid behavior. Here, m_D is a cut-off factor and should be an order of the Debye mass. Differentiating Eq.(14) with respect to p, we obtain $\frac{dp(\omega)}{d\omega}$ at leading order in $\frac{T}{\mu}$ is

$$\begin{aligned} \frac{dp(\omega)}{d\omega} &\simeq \left(1 - \frac{\partial}{\partial\omega}\mathrm{Re}\Sigma(\omega)\right)\frac{E_p(\omega)}{p(\omega)} \\ &= \left[1 + \frac{C_F\alpha_s}{3\pi}\ln\left(\frac{m_D}{T}\right)\right]\frac{E_p(\omega)}{p(\omega)} \end{aligned} \tag{16}$$

where α_s is the strong coupling constant, $C_F = (N_c^2 - 1)/(2N_c)$ and N_c is the color factor. Using Eqs.(16), (9) and (11-13), the neutrino mean free path can be determined for two conditions. For $|p_f(u) - p_f(e)| \geq |p_f(d) - p_f(\nu)|$

$$\frac{1}{l_{mean}^{abs,D}} = \frac{4}{\pi^3} G^2 \cos^2\theta_c \frac{\mu_u^2 \mu_e^3}{\mu_\nu^2} \Big[1 + \frac{1}{2}\Big(\frac{\mu_e}{\mu_u}\Big) + \frac{1}{10}\Big(\frac{\mu_e}{\mu_u}\Big)^2\Big]$$
$$\times\ [(E_\nu - \mu_\nu)^2 + \pi^2 T^2]\Big[1 + \frac{C_F \alpha_s}{3\pi} \ln\Big(\frac{m_D}{T}\Big)\Big]^2 \tag{17}$$

To derive above Eq.(17), we use the result of integral [8, 32]

$$\int_0^\infty dE_d \int_0^\infty dE_u \int_0^\infty dE_e [1 + e^{-\beta(E_\nu - \mu_\nu)}] n(p_d)[1 - n(p_u)][1 - n(p_e)]\delta(E_d + E_\nu - E_u - E_e)$$
$$\simeq \frac{1}{2}[(E_\nu - \mu_\nu)^2 + \pi^2 T^2] \tag{18}$$

Similarly, for $|p_f(d) - p_f(\nu)| \geq |p_f(u) - p_f(e)|$, the corresponding expression for mean free path can be obtained by replacing $\mu_u \leftrightarrow \mu_d$ and $\mu_e \leftrightarrow \mu_\nu$ in Eq.(17). Since quark and electron are assumed to be massless, the chemical equilibrium condition gives $p_f(u) + p_f(e) = p_f(d) + p_f(\nu)$, which we use to derive Eq.(17).

The other major contribution to the mean free path arises from quark-neutrino scattering. The neutrino scattering process from degenerate quarks are given by

$$q_i + \nu_e(\overline{\nu}_e) \rightarrow q_i + \nu_e(\overline{\nu}_e) \tag{19}$$

for each quark component of flavor $i(= u$ or $d)$. The scattering mean free path of the neutrinos in degenerate case can be calculated similarly as evaluated by Lamb and Pethick in [23] for electron-neutrino scattering. Assuming $m_{q_i}/p_{f_i} \ll 1$ and including the non-Fermi liquid correction through phase space, the mean free path is given by

$$\frac{1}{l_{mean}^{scatt,D}} = \frac{3}{16} n_{q_i} \sigma_0 \Big[\frac{(E_\nu - \mu_\nu)^2 + \pi^2 T^2}{m_{q_i}^2}\Big]\Big[1 + \frac{C_F \alpha_s}{3\pi} \ln\Big(\frac{m_D}{T}\Big)\Big]^2 \Lambda(x_i) \tag{20}$$

Here, m_{q_i} is the quark mass. C_{V_i} and C_{A_i} is the vector and axial vector coupling constant was given in TABLE-II of Ref.[8]. In Eq.(20) if we drop the color factor and the second square bracketed term, we obtain results reported in [23] for dense and cold QED plasma with m_q replaced by m_e. In Eq.(20) the constants $\sigma_0 \equiv 4G^2 m_{q_i}^2/\pi$ [22] and n_{q_i} is the number density of quark, is given by

$$n_{q_i} = 2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(E_{q_i} - \mu_{q_i})}} \tag{21}$$

where 2 is the quark spin degeneracy factor. Explicit form of $\Lambda(x_i)$ can be written as [8, 23]

$$\Lambda(x_i) \;=\; \frac{4}{3}\frac{\mathrm{Min}(\mu_\nu, \mu_{q_i})}{\mu_{q_i}}\Big[(C_{V_i}^2 + C_{A_i}^2)\Big(2 + \frac{1}{5}x_i^2\Big) + 2C_{V_i}C_{A_i}x_i\Big] \tag{22}$$

and $x_i = \mu_\nu/\mu_{q_i}$ if $\mu_\nu < \mu_{q_i}$ and $x_i = \mu_{q_i}/\mu_\nu$ if $\mu_\nu > \mu_{q_i}$.

B. Nondegenerate Neutrinos

We also derive mean free path for nondegenerate neutrinos *i.e.* when $\mu_\nu \ll T$. For nondegenerate neutrinos the inverse process (6) is dropped. Hence we neglect the second term in the curly braces of Eq.(7). In this case, only those fermions whose momenta lie close to their respective Fermi surfaces can take part in a reaction. It is to be mentioned here, if quarks are treated as free, as discussed in [8, 33, 34], the matrix element vanishes, since u, d quarks and electrons are collinear in momenta. The inclusion of strong interactions between quarks relaxes these kinematic restrictions resulting in a nonvanishing squared matrix amplitude. Since the neutrinos are produced thermally, we neglect the neutrino momentum in energy-momentum conservation relation [8]. This is not the case for degenerate neutrinos where $p_\nu \gg T$ and therefore such approximation is not valid there. By doing angular average over the direction of the outgoing neutrino, from Eq.(8) the squared matrix element is given by [9]

$$\begin{aligned}|M|^2 \;&=\; 64G^2\cos^2\theta_c p_f^2(V_d\cdot P_\nu)(V_u\cdot P_e)\\ &=\; 64G^2\cos^2\theta_c p_f^2 E_\nu\mu_e\frac{C_F\alpha_s}{\pi}\end{aligned} \tag{23}$$

where $V = (1, v_f)$ is the four velocity. To calculate Eq.(23) we have used the chemical equilibrium condition $\mu_d = \mu_u + \mu_e$ and also the relations derived from Fermi liquid theory are given by[9]

$$v_F = 1 - \frac{C_F\alpha_s}{2\pi}\;; \qquad \delta\mu = \frac{C_F\alpha_s}{\pi}\mu \tag{24}$$

Putting $|M|^2$ in Eq.(7) we have,

$$\begin{aligned}\frac{1}{l_{mean}^{abs,ND}} \;&=\; \frac{3C_F\alpha_s}{4\pi^6}G^2\cos^2\theta_c\int d^3p_d\int d^3p_u\int d^3p_e\\ &\quad\delta^4(P_d + P_\nu - P_u - P_e)n(p_d)[1-n(p_u)][1-n(p_e)]\end{aligned} \tag{25}$$

Neglecting the neutrino momentum in the neutrino momentum conserving δ function, the integrals can be decoupled into two parts. Following the procedure of Iwamoto [8], the angular integral is given by

$$\mathcal{A} = \int d\Omega_d \int d\Omega_u \int d\Omega_e \delta(p_d - p_u - p_e) = \frac{8\pi^2}{\mu_d \mu_u \mu_e} \tag{26}$$

and the other part

$$\begin{aligned}\mathcal{B} = & \int_0^\infty p_d^2 \frac{dp_d}{dE_d} dE_d \int_0^\infty p_u^2 \frac{dp_u}{dE_u} dE_u \int_0^\infty p_e^2 dE_e \\ & \delta(E_d + E_\nu - E_u - E_e) n(p_d)[1 - n(p_u)][1 - n(p_e)]\end{aligned} \tag{27}$$

Changing the variables to $x_d = (E_d - \mu_d)\beta$, $x_u = -(E_u - \mu_u)\beta$ and $x_e = -(E_e - \mu_e)\beta$ and denoting the single particle energy $E_{u(d)}$ as ω we have from Eq.(27)

$$\mathcal{B} = \int_{-\infty}^\infty dx_d dx_u dx_e \frac{dp_d(\omega)}{d\omega} \frac{dp_u(\omega)}{d\omega} p_d^2 p_u^2 p_e^2 \delta(x_d + x_u + x_e + \beta E_\nu) n(x_d) n(-x_u) n(-x_e) \tag{28}$$

As the contribution dominates near the Fermi surfaces, extension of lower limit is a reasonable approximation [32, 35].

Using Eq.(16) and perform the integration of Eq.(28) following the procedure defined in [16, 32, 35], we have

$$\mathcal{B} = \mu_d^2 \mu_u^2 \mu_e^2 \frac{(E_\nu^2 + \pi^2 T^2)}{2(1 + e^{-\beta E_\nu})} \Big[1 + \frac{C_F \alpha_s}{3\pi} \ln\Big(\frac{m_D}{T}\Big)\Big]^2 \tag{29}$$

Using Eqs.(25), (26) and (29), the mean free path at leading order in T/μ is given by

$$\frac{1}{l_{mean}^{abs,ND}} = \frac{3C_F \alpha_s}{\pi^4} G^2 \cos^2\theta_c \; \mu_d \; \mu_u \; \mu_e \; \frac{(E_\nu^2 + \pi^2 T^2)}{(1 + e^{-\beta E_\nu})} \Big[1 + \frac{C_F \alpha_s}{3\pi} \ln\Big(\frac{m_D}{T}\Big)\Big]^2 \tag{30}$$

The first term is known from [8] and the additional terms are higher order corrections to the previous results derived in the present work.

For the scattering of nondegenerate neutrinos in quark matter, the expression of mean free path was given by Iwamoto[8]. We incorporate the anomalous effect which enters through phase space modification giving rise to

$$\frac{1}{l_{mean}^{scatt,ND}} = \frac{C_{V_i}^2 + C_{A_i}^2}{20} n_{q_i} \sigma_0 \Big(\frac{E_\nu}{m_{q_i}}\Big)^2 \Big(\frac{E_\nu}{\mu_i}\Big) \Big[1 + \frac{C_F \alpha_s}{3\pi} \ln\Big(\frac{m_D}{T}\Big)\Big]^2. \tag{31}$$

Here, we have assumed $m_{q_i}/p_{f_i} \ll 1$ and the constants σ_0 and number density n_{q_i} defined earlier.

III. RESULTS AND CONCLUSION

Armed with the results of the previous sections, we now estimate the numerical values of the neutrino mean free paths. Here, E_ν is set to be equal to $3T$ and $m_q = 10$ MeV[8, 24]. For the quark chemical potential, following ref.[9], we take $\mu_q \simeq 500$ MeV corresponding to densities $\rho_b \approx 6\rho_0$, where ρ_0 is the nuclear matter saturation density. The electron chemical potential is determined by using the charge neutrality and beta equilibrium conditions which yields $\mu_e = 11$ Mev. The other parameters used are same as [9].

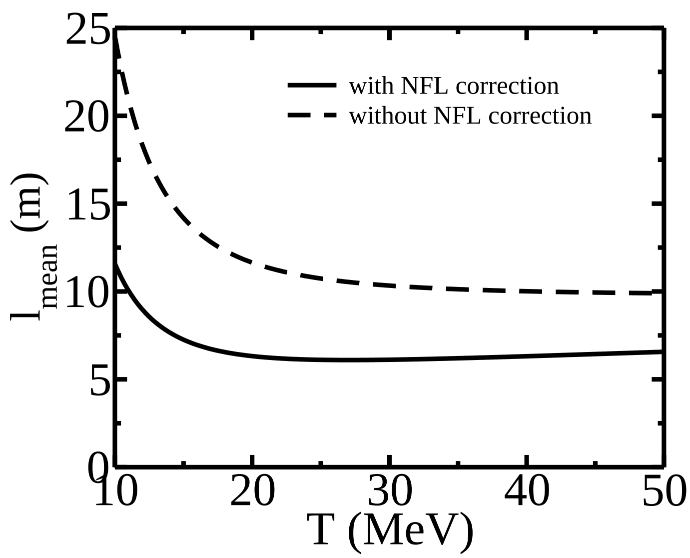

FIG. 1: Neutrino mean free path in quark matter for degenerate neutrinos.

From fig (1), we find for degenerate neutrinos the anomalous logarithmic terms reduces the value of the mean free path appreciably both in the low and high temperature regime. Fig (2) shows that for nondegenerate neutrinos NFL corrections is quite large at low temperature while at higher temperature it tends to merge. It is interesting to see from these two plots, that NFL corrections to the MFP in degenerate neutrinos is less than that of nondegenerate neutrinos. This reduced mean free path is expected to influence the cooling of the compact stars. It is also to be noted that all the results presented above are restricted to the leading log approximation which can be extended to take the next to leading order corrections into account [36]. We plan to undertake such investigations in a future publication [37].

Acknowledgments

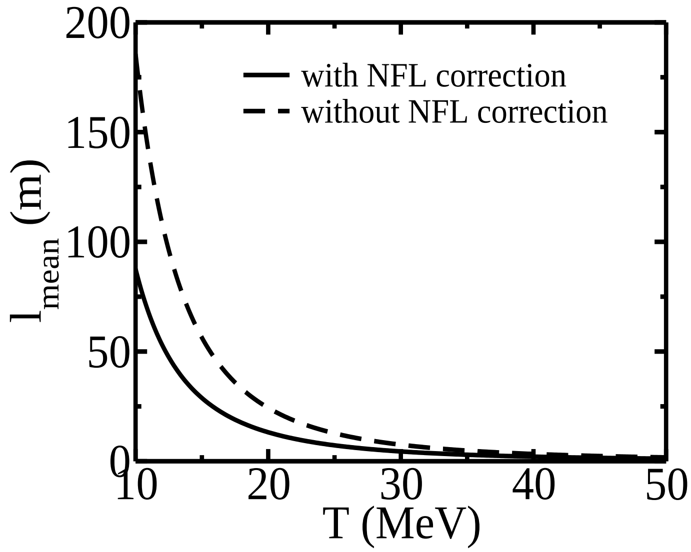

FIG. 2: Neutrino mean free path in quark matter for nondegenerate neutrinos.

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